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Long run control of Markov processes with degenerate observation

We are given a controlled discrete time Markov process (x_n) with transition operator $P^{u_n}(x_n, \cdot)$ at generic time n, and observation of the form $y_n = h(x_n)$. We assume that the observation space Y is at most countable and controls u_n at time n are adapted to the observation σ - field $Y^n = \sigma\{y_0, y_1, \ldots, y_n\}$. Our purpose first is to maximize long run cost functional $J_\mu(V) = \liminf_{n\to\infty} \frac{1}{n} E_\mu \left\{ \sum_{i=0}^{n-1} c(x_i, u_i) \right\}$, where c is a running reward. The problem has been solved in the papers [1] and [2] under the assumption that the information structure is corrupted with an additive noise. The method used in particular in [2] was based on reference probability i.e. introduction an equivalent probability measure under which the state and observation processes were independent. In the case of degenerate observation, as above, this method does not work. Another alternative was to study ergodicity of controlled filtering processes. Examples however based on the paper [3] show that even when the state process has nice ergodic properties the filtering process may have many invariant measures. The result solves the problem using a generalization of the convexity method considered in [1]. Further step is to solve long run risk sensitive problem with degenerate observation generalizing paper [4]. Completely open problem is to study the case with uncountable state space Y.

References:

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