

TITLE: Extension of the Itô-Nisio theorem, strong pathwise convergence in series expansions of Lévy processes, and the continuity of Itô map

SPEAKER: Jan Rosiński

AFFILIATION: University of Tennessee, Knoxville

ABSTRACT: The Itô-Nisio theorem is a classical tool to show the pathwise uniform convergence in the Karhunen-Loève-type expansion of a continuous Gaussian process over a compact index set. In a more general context, Itô-Nisio theorem holds for series of independent random elements in a separable Banach space. However, the assumption of separability limits applications of this theorem, since many stochastic processes have sample paths in non-separable Banach spaces, such as $D[0, 1]$ (under supremum norm), the space H_α of α -Hölder continuous functions, the space BV_p of càdlàg functions of bounded p -variation, etc. Since the convergence of series expansions in the norms of such spaces is often desirable, we pursue the non-separable extension of this theorem.

In this talk, we plan to cover the following topics:

- Provide the proper framework for the Itô-Nisio theorem in non-separable Banach spaces and give sufficient conditions on Banach spaces under which such theorem holds;
- Discuss examples of function spaces, relevant to probability, where the Itô-Nisio theorem holds or does not hold, including BV_ϕ and the Wiener space BV_ϕ^0 ;
- Deduce the a.s. convergence of the Karhunen-Loève-type expansions of Lévy processes in appropriate BV_ϕ spaces;
- Establish the continuity of Itô map, when the Lévy process, as an input, is approximated by pice-wise linear processes of its Karhunen-Loève-type expansion.

This talk is based on a joint work with Andreas Basse-O'Connor and Jørgen Hoffmann-Jørgensen of Aarhus University.