Finite-time blowup of the Fujita equation with fractional Laplacian perturbed by fractional Brownian motion

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Abstract

We provide conditions implying finite-time blowup of positive weak solutions to the SPDE

$$\begin{aligned} du(t,x) &= \left[\Delta_\alpha u(t,x) + Ku(t,x) + u^{1+\beta}(t,x)\right] dt + \mu u(t,x)\, dB_t^H, \\ u(0,x) &= f(x),\ x \in \mathbb{R}^d,\ t \geq 0, \end{aligned} \tag{1}$$

where $\alpha \in (0,2]$, $K \in \mathbb{R}$, $\beta > 0$, $\mu \geq 0$ and $H \in [\frac{1}{2}, 1)$ are constants, Δ_α is the fractional power $-(-\Delta)^{\alpha/2}$ of the Laplacian, (B_t^H) is a fractional Brownian motion with Hurst parameter H, and $f \gneq 0$ is a bounded measurable function. To achieve this we investigate the a.s. finiteness of exponential functionals of the form

$$\int_{r_0}^{\infty} \left[\frac{e^{Ks+\mu B_s^H}}{s^{d/\alpha}}\right]^{\beta} ds$$

with $r_0 > 0$. Moreover, we obtain bounds for the explosion times of the equation above in the cases $K < 0$ and $K > 0$. Setting $K = \mu = 0$ in (1) we recover the classical deterministic result proved by Sugitani (1975) that, if $d \leq \alpha/\beta$, then the equation $\frac{\partial}{\partial t}u(t,x) = \Delta_\alpha u(t,x) + u^{1+\beta}(t,x)$ in $\mathbb{R}^d$ possesses no non-trivial positive global solutions.