Henryk Gacki
Institute of Mathematics
University of Silesia in Katowice, Poland

Asymptotic stability of an evolutionary nonlinear Boltzmann-type equation

Some problems of the mathematical physics can be written as differential equations for functions with values in the space of measures. Unfortunately, the vector space of signed measures doesn't have good analytical properties (see e.g. the Fortet-Mourier metric is not complete is such space).

There are two methods to overcome this problem. First, we may replace the original equations by the adjoint ones on the space of continuous bounded functions. Secondly, we may restrict our equations to some complete convex subsets in the vector space of measures. The last approach seems to be quite natural and is related to the classical results concerning differential equations on convex subsets of Banach spaces (see [2]). The convex sets method in studying the Boltzmann equation was used in a series of papers (see for example : [1, 3, 5, 6, 7]).

The main purpose of the talk is to show some application of the Kantorovich-Rubinstein maximum principle concerning the properties of probability metrics. Namely, we show that the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle used in the theory of dynamical systems allow us to find new sufficient conditions for the asymptotic stability of solutions of a version of the nonlinear Boltzmann-type equation. Such equation was stimulated by the problem of the stability of solutions of the following version of the Boltzmann equation (see [3])

$$\frac{d\psi}{dt} + \psi = P\,\psi \qquad \text{for} \qquad t \geq 0 \tag{1}$$

with the initial condition

$$\psi\,(0) = \psi_0, \tag{2}$$

where $\psi_0 \in \mathcal{M}_1(\mathbb{R}_+)$ and $\psi : \mathbb{R}_+ \to \mathcal{M}_{sig}(\mathbb{R}_+)$ is an unknown function, while P is the collision operator acting on the space of probability measures.

We will discuss an equation drawn from the kinetic theory of gases, which will be a generalized version of (1) (see [4]) in the sense that the collision operator P will be a convex combination of N operators $P_1, ..., P_N$, with P_k for $k \geq 2$ describing the simultaneous collision of k particles and P_1 being the influence of external forces.

The same equation which was discussed by Lasota [5] and his stability result was based on the technique of Zolotarev metrics. Lasota showed that the stationary solution is exponentially stable in the Zolotarev norm of order 2.

In our paper we show, using the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle, that if our equation has a stationary measure $\mu_\star$ such that supp$\mu_\star = \mathbb{R}_+$, then this measure is asymtotically stable with respect to the Kantorovich-Wasserstein metric.

An open problem related with characterization of the stationary measure $\mu_\star$ of the equation (1) will be presented at the end. The talk is based on a joint work with Łukasz Stettner from Institute of Mathematics of PAS.

References

[1] R. Brodnicka and H. Gacki: Asymptotic stability of a linear Boltzmann-type equation, Appl. Math., 41 (2014), 323–334.

[2] M. G. Crandall, *Differential equations on convex sets*, J. Math. Soc. Japan 22, 443–455, (1970).

[3] H. Gacki, *Applications of the Kantorovich–Rubinstein maximum principle in the theory of Markov semigroups*, Dissertationes Math., 448, 1–59, (2007).

[4] H. Gacki and Ł. Stettner, *Asymptotic stability of some nonlinear evolutionary equation*, in preparation.

[5] A. Lasota, *Asymptotic stability of some nonlinear Boltzmann-type equations*, J. Math. Anal. Appl. 268 (2002), 291-309.

[6] A. Lasota and J. Traple, *Asymptotic stability of differential equations on convex sets*, J. Dynam. Differential Equations 15, 335–355, (2003).

[7] A. Lasota and J. Traple, *Properties of stationary solutions of a generalized Tjon–Wu equation*, J. Math. Anal. Appl. 335, 669 – 682, (2007).